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## Notes on the determination of the angular subtense of the apparent source in laser safety

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# Notes on the Determination of the Angular Subtense of the Apparent Source in Laser Safety

Paper # 1205

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## Abstract

In the wavelength range of 400 nm to 1400 nm, the retinal thermal exposure or emission limits depend on the angular subtense of the apparent source (Greek symbol “alpha”). For the case that a given laser beam is associated to an extended source, according to IEC 60825-1 or ANSI Z136.1 the emission permitted for a given class (such as Class 1) can be substantially higher as compared to the case of a small source. In this paper, earlier discussions on the classification concept to analyse extended sources are summarized and commented. While it is historically justified to refer to the “apparent source” such as when the optical source is a diffusor as the classical example of an extended source, the more general understanding and terminology is to associate “alpha” with the angular subtense of the retinal image, i.e. the irradiance profile on the retina. This is particularly important when the aperture stop on the imaging system (the eye) reduces significantly the retinal image as compared to the angular subtense of the source, which is possible for coherent and partially coherent beams. In a second part of the paper, modelling results for the potential impact of the aperture stop to reduce the retinal image are discussed for the example of two partially coherent beams.

## Scope

This proceeding paper is structured into two parts. In the first part, a short review and comments are provided on earlier papers of the Seibersdorf Laboratories group on the topic of the “apparent source” in laser safety. In the second part, the effect of an aperture stop on the retinal image (and therefore on the value of  $\alpha$ , the angular subtense of the apparent source) is discussed for partially coherent laser beams (i.e.  $M^2 > 1$ ) by way of two example beams.

## Introduction

For the retinal thermal limits, the default condition for collimated laser beams is a “small source” or “point source” where for accommodation to infinity, the laser radiation that enters the eye forms a minimum retinal image. In this case, the retinal thermal safety limits of

IEC 60825-1 [1], ANSI Z136.1 [2] and ICNIRP [3] do not depend on the diameter of the retinal image. The limits and the safety analysis are correspondingly simple. For some special types of radiation sources, however, laser products and exposures can be associated to “extended sources” when the retinal irradiance profile is larger than  $\alpha_{\min}$  which equals 1.5 mrad. Special limit tables and rules for the determination of the value of  $\alpha$  are given in the standards. The extended source concept presented in IEC 60825-1 Edition 3 as well as already for Edition 2, for the classification of products, has been developed between 1999 and 2005 based on beam propagation modelling (more details on the history can be found in the ILSC 2015 paper [4]).

In the following, the abbreviation AE refers to Accessible Emission and AEL refers to Accessible Emission Limit, implied to be for Class 1, 1M, 2, 2M or 3R since for these AEL, retinal thermal limits are defined.

The concept which was adopted in the second edition of IEC 60825-1 was discussed in an ILSC 2005 paper [5] and the main points are (the discussion is for measurement Condition 3; for measurement condition 1 equivalent concepts apply):

- For the general classification of extended sources it is necessary to consider positions of the aperture stop in the beam that are further than 100 mm from the beam waist (or whatever is specified as reference point).
- For each position of the aperture stop in the beam (representing the pupil of the eye), the retinal image has to be determined for varying accommodation conditions, from 100 mm in front of the “eye” to infinity.
- When the beam at the aperture stop (i.e. at the pupil of the eye) is smaller than the aperture stop, then the retinal image is the conjugate of the irradiance profile that is present at the location of accommodation. For instance, when the eye accommodates to 200 mm in front of the eye, the retinal image (the retinal irradiance distribution) is equal to the irradiance profile (but scaled in size as

dictated by the lens makers formula) present at 200 mm; the “object” that is imaged can also be an irradiance profile in virtual space, such as “behind” a lens. When the eye accommodates to infinity, the irradiance distribution in the image plane reflects the angular distribution of the beam in the far field, or in other words, the divergence (but again, potentially truncated by the aperture stop). We note here that the potential influence of the aperture stop on the retinal image was discussed and highlighted in the 2015 ILSC paper (demonstrated with the example of three collimated beams crossing over) but was not so in the earlier 2005 paper.

- The most restrictive position and the most restrictive accommodation, which is the one resulting in the highest ratio of AE/AEL, has to be used for classification. In other words, when the AE is smaller than the AEL (such as of Class 1) for all relevant positions in the beam and for all relevant accommodation conditions, then the product can be classified with the respective class.
- For beams with a beam quality factor  $M^2 > 1$  (i.e. higher order beams), the 2<sup>nd</sup> moment diameter, determined according to ISO 11146 [6] in many cases cannot be used to determine the value of  $\alpha$ . Examples of gross deviations are discussed in Reference [7]. IEC 60825-1 Edition 2 and Edition 3 as well as ICNIRP 2013 define a general method to determine  $\alpha$  for arbitrary image irradiance profiles. The method is also discussed in the 2015 ILSC paper [4]. It is noted that this “image analysis” method is based on variation of the size and position of a field stop and determination of the maximum ratio of AE through the field stop over the AEL. Thereby the image analysis method does not only determine the value of  $\alpha$ , but also has an impact on the AE.
- Also in terms of calculating the power passing through the aperture stop, the 2<sup>nd</sup> moment diameter of the beam incident on the aperture stop can grossly err on the unsafe side [7]. Consequently, while the application of beam propagation theory was very instructive to understand the issue and develop a general concept to classify extended sources which is in place since Edition 2 of IEC 60825-1, the beam propagation model developed at the end of the 20<sup>th</sup> century (with the 2<sup>nd</sup> moment beam parameters as the input and the output of  $\alpha$  and the partial power passing through the aperture stop), for the case of  $M^2 > 1$  has little quantitative value. Either more complex modelling methods are needed or the classification has to be based on direct measurements of the

retinal image profile (with a lens and a CCD camera).

Per position of the “eye” in the beam, the location of accommodation (where the “eye” is looking at) that produces the most restrictive retinal image (the largest AE/AEL) can be referred to as the “location of the apparent source”. In some cases, however, the location of the apparent source depends on position in the beam, which means that for a given spatial laser emission (for a given beam), there is not *ONE* apparent source associated to that emission. This was noted in the 2005 ILSC paper as one of the misnomers of “The apparent source”. In that paper it was also commented that the reference to the “source” is often misleading as it is more generally the *retinal image* which needs to be considered. This is particularly the case when the 7 mm aperture stop has an influence on the image profile so that the image is smaller than the irradiance profile where the eye is accommodating to, because not all rays that make up the beam that is incident on the eye are available to form the retinal image.

Beam propagation modelling showed [4, 5, 8] that high quality beams ( $M^2 = 1$  and in the lower  $M^2$  range) that are circularly symmetric and that are not scanned always represent a “small source”, i.e. are always associated to  $\alpha = \alpha_{\min}$ . We note that for  $M^2 = 1$ , the beam parameter product is minimum and equals

$$\theta \cdot d_0 = \frac{4\lambda}{\pi} \quad (1)$$

where  $\theta$  is the full divergence and  $d_0$  is the diameter of the beam waist, both determined at the  $1/e^2$  irradiance points of the Gaussian profile. Therefore, when the divergence is large, the beam waist diameter becomes correspondingly small; when the beam waist diameter is large, the divergence is very small. For the wavelength range of 400 nm to 1400 nm, the result is that accommodation to either the beam waist or to infinity is associated to an angular subtense of the apparent source less than  $\alpha_{\min}$  (even when potential truncation of the beam is not considered). This also applies to somewhat higher  $M^2$  values of for instance up to  $M^2 = 29$  for a wavelength of 532 nm and up to  $M^2$  of 14 for a wavelength of 1064 nm [4] assuming here that the 2<sup>nd</sup> moment is appropriate for the determination of  $\alpha$  (which is often not the case). This result was obtained for neglecting truncation effects of the aperture on the image profile, i.e. assuming that the beam diameter at the pupil is smaller than the pupil. Truncation effects, such as described in another ILSC paper [9] for zero order beams ( $M^2 = 1$ ) and further below for examples of higher order beams make the retinal image even smaller as predicted by the beam propagation model.

However, for two cases can low quality factors ( $M^2=1$ ) still be associated to an extended source: first, when the beam is astigmatic (such as a line laser), second, when the beam is scanned (either astigmatic beams or stigmatic beams). Also in some cases, an assembly of several beams or arrays can be an “extended source”.

For a scanned beam, only accommodation to the pivot point of the scanned beam (i.e. usually the mirror) produces a stationary image on the retina (see for instance figure 4.13 in Henderson and Schulmeister [8]). Accommodation to other positions than the mirror produces a retinal image that is moving, which often is less restrictive as compared to accommodation to the mirror. With a finite extent of the beam profile at the mirror, often an extended source results. For the classification of a scanned product, however, it is necessary to also consider distances further than 100 mm from the pivot point, that result in a smaller angular subtense of the retinal image of the beam profile that is present at the mirror so that the most restrictive position is often further than at 100 mm from the pivot point.

The second case where a high quality beam can be associated to a value of  $\alpha$  greater than  $\alpha_{\min}$  are astigmatic beams where the beam waist is at different locations for the two beam axis. For instance, the beam could feature relatively high divergencies and small beam waist diameters for both axis, so that when the beam were circularly symmetric and the beam waists would be at the same position, accommodation to the beam waist would result in a small source. However, when the beam waist positions are separated, then accommodation to one of the beam waists produces a minimum extent only on one axis, in the other axis the beam diameter at that position is correspondingly larger. The eye can only accommodate to one of the two beam waists at a given time, so that this can be an example where also at the most restrictive position,  $\alpha$  (as arithmetic mean of the two axis) is greater than  $\alpha_{\min}$ . A classic example for a highly astigmatic beam is a line laser, with a well collimated direction to form the “thickness” of the line. The beam waist in the strongly diverging direction is in or very close to the line shaping optics. Accommodation to this beam waist also usually produces the smaller overall (average)  $\alpha$ : while in the axis of the large divergence this beam waist is often resulting in a minimum angular subtense for the respective “width” of this beam waist, in the other axis (the collimated), the line “thickness” results in an extended apparent source. A line laser is also a good example where it can be understood (and confirmed by looking into the beam) that the aperture stop (representing the pupil of the eye) truncates the beam as it enters the eye. When the eye accommodates to infinity

(see also figure 4.10 in Henderson and Schulmeister [8]) a sharp line results as retinal image. However, the length of the line is a direct result of the truncation by the pupil of the eye and is equal to the angle subtended by the pupil (such as 7 mm diameter) as subtended from location of the beam waist. That is for the position of the eye at 100 mm from the beam waist in the highly diverging direction, the length of the line on the retina equals  $7 \text{ mm} / 100 \text{ mm} = 70 \text{ mrad}$  for a 7 mm pupil. The respective borderline rays are shown in Figure 1 for the example of two diameters of the aperture in the beam.

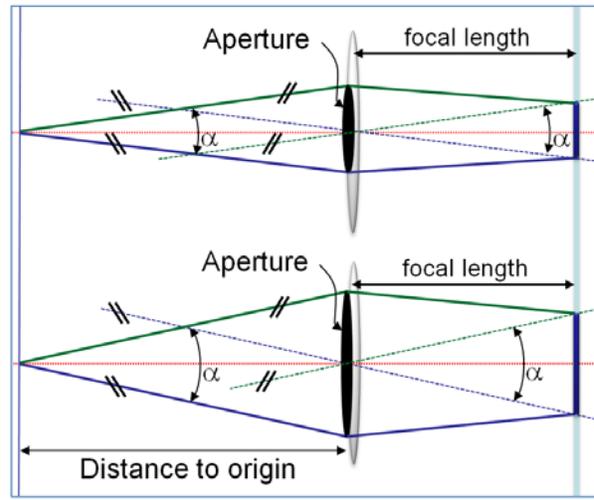


Figure 1. Schematic drawing to demonstrate that for accommodation to infinity (image plane is in focal plane of lens), for an origin of the rays at some finite distance in front of the eye (such as typical for the highly diverging axis of a line laser) the angular subtense of the retinal image is equal to the angle subtended by the aperture as seen from the origin.

There are several typical cases where higher order beams represent an extended source, but it is a challenge to determine the accessible emission and the retinal image accurately with a model. The 2<sup>nd</sup> moment method of ISO 11146 certainly does not lend itself, as discussed above and in reference [7].

The classic example of an extended source is a diffusor that is irradiated with a laser beam so that the transmitted or reflected optical radiation forms, when of sufficient diameter, an extended source. The radiation is completely scattered so that the resulting radiation is spatially incoherent. Each point of the diffusor can be envisaged as to emit a spherical wave. For accommodation to the diffuse surface, the wavefronts from all points that make up the irradiated diffuse surface are incident on the pupil and parts of them also pass into the eye to make up the retinal image. The wavefronts that were emitted from each point on the diffusor converge to a conjugate point to form the retinal

image. Since radiation from all points pass through the pupil and the wavefronts are homogeneously mixed at the position of the pupil, the diameter of the pupil does not have an influence on the size of the image. The size of the pupil has an influence on the absolute level of the irradiance of the image, but the image shape does not depend on the pupil diameter. We also know this from our general experience with vision: our pupil varies frequently in diameter but the image that we see does not change. However, this is the case as a general rule only for images formed by incoherent radiation. For coherent or partially coherent beams, the “information” of the light field is not evenly distributed across the pupil but is localised, so that the pupil can truncate (block) some of the rays that make up the beam. These rays are then missing to form the image, resulting in a smaller image as compared to a larger pupil where the whole beam passes through the pupil. A very instructive example was shown in the ILSC 2015 paper [4] for the example of three collimated beams where for accommodation to infinity the number of spots on the retina depend on how many of the beams pass through the pupil. For accommodation to the cross-over point, when the beams have the same diameter there, it does not matter if one or all three beams pass into the eye, the image has the same diameter – we will see that this situation has a high equivalence to the partially coherent beams discussed in the next section.

### Example of Impact of Aperture Stop on Retinal Image for Partially Coherent Beams

#### Basics

As noted above, and as we know from experience, for completely spatially incoherent sources such as originating from diffuse transmission or reflection (an example for the first is a frosted light bulb, an example for the latter is the moon), the image of that source does not change in size when the pupil of the eye constricts or dilates. For coherent or partially coherent beams, however, the pupil of the eye (or the 7 mm aperture stop for the classification of laser products) can result in a smaller image as would be the case for an imaging system where the aperture stop is larger than the beam, so that the full beam enters the imaging system. Therefore, only for a beam that is smaller than 7 mm can it be generally said that the irradiance profile of the retinal image is equal (scaled by the usual difference between the object and the image) to the irradiance profile in the beam where the eye is accommodating to (the “object” can also be a virtual beam profile). When the beam is larger than the pupil, some rays that otherwise make up the image are lost and in that case the retinal image is correspondingly smaller. This is relevant, because a smaller retinal image of the apparent

source  $\alpha$  results in smaller permitted (safe) emission levels for the laser.

A fully spatially coherent Gaussian beam (i.e. a diffraction limited Gaussian beam or TEM<sub>00</sub> mode) has a fixed relationship between the beam waist diameter and the divergence of the beam (equation 1 above). A partially coherent beam (also sometimes referred to as poor beam quality) has, for the same beam waist diameter, a larger divergence as compared to the fully coherent beam.

The ratio between the beam parameter product (waist diameter  $\times$  divergence) of the partially coherent beam and the beam parameter product of a diffraction limited Gaussian beam is referred to as  $M^2$ , the beam quality factor:

$$M^2 = \theta \cdot d_0 \frac{\pi}{4\lambda} \quad \text{or} \quad (2)$$

$$\theta = M^2 \cdot \frac{4\lambda}{d_0\pi} \quad (3)$$

where  $\theta$  is the full angle divergence and  $d_0$  is the diameter of the beam waist determined according to the 2<sup>nd</sup> moment method defined in ISO 11146 [6].

The Rayleigh length  $z_R$  is defined as the distance from the beam waist, where the beam diameter in the respective axis is a factor of the square-root of 2 larger than the beam waist (for stigmatic beams, the area is a factor 2 larger at the Rayleigh length). For a “lower quality” beam with  $M^2 > 1$ , the Rayleigh length is smaller, since for a given beam waist diameter, the divergence is larger by the factor  $M^2$ . Although the formula for the Rayleigh length is usually given in a different way, it is simplest to define and understand the Rayleigh length as the ratio of the beam waist diameter to the full angle divergence:

$$z_R = \frac{d_0}{\theta} \quad (4)$$

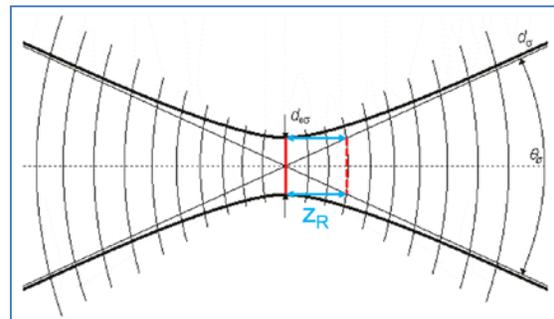


Figure 2. Schematic drawing of the region around the beam waist with a given diameter. The Rayleigh length  $z_R$  is shown. It is the distance from the beam waist where the diameter of the beam (determined with the 2<sup>nd</sup> moment method for non-Gaussian beam profiles) is a factor of the square-root two larger than the beam waist. The Rayleigh length is also equal to the ratio of beam waist diameter over the divergence.

We see graphically in Figure 2 that the Raleigh length is that distance from the beam waist where the straight lines that are the asymptotes to the far field beam envelope are a distance  $d_0$  apart (this is a logical relationship, since the angle defined by those asymptotes is the divergence  $\theta$  and from equation (4) we see that  $z_R \cdot \theta = d_0$ ). This is a simple way to graphically determine the Raleigh length (often schematical drawings are not correct in this respect, for instance the entry “Rayleigh Length” in Wikipedia.org (retrieved 5<sup>th</sup> February 2019).

When we insert the equation for  $M^2$  (equation (2)) into the equation for the Raleigh length (equation (4)), we obtain the form of the Raleigh length more often seen in the literature:

$$z_R = \frac{1}{M^2} \frac{\pi d_0^2}{4\lambda} \quad (5)$$

### Gauss-Schell Beam as Model

Further below, results of model calculations for the impact of apertures on the retinal image are presented for partially coherent beams. The partially coherent beams are characterised by a larger beam parameter product as compared to the fully coherent Gaussian beam, i.e.

$$\theta \cdot d_0 = M^2 \cdot \frac{4\lambda}{\pi} \quad (6)$$

The influence of the aperture “cutting” some part of the partially coherent beam away is modelled in the following way: the partially coherent beam is described and modelled as a Gauss-Schell beam [10, 11] where the larger  $M^2$  is a result of (only) partial incoherence, and the beam irradiance profile is Gaussian along the beam up to the aperture. The overall, partially coherent beam can in this model be made up by superposition of fully coherent (i.e.  $M^2 = 1$ ) Gaussian beams that together result in an irradiance profile that has the beam waist diameter and divergence that is defined for the partially coherent beam. To obtain the retinal image profile, geometrical optical principles are applied, which are a good approximation when the radiation field has a low degree of spatial coherence.

The Gauss-Schell beam (GSB) can be composed of fully coherent beams in a number of ways, where the following two are instructional to envisage and understand the results of the model shown further below. The partially coherent beam which is modelled as Gauss-Schell beam can as one option (Figure 3) be “constructed” by a superposition of fully coherent

beams ( $M^2 = 1$ ) that have the same beam waist diameter as the GSB but a correspondingly smaller divergence (smaller by a factor  $M^2$ ).

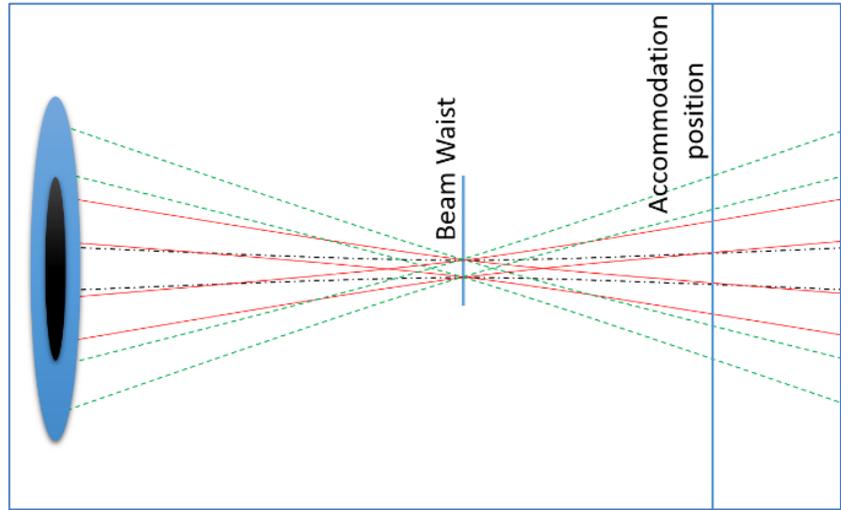


Figure 3. One way to construct a Gauss-Schell Beam is to superimpose fully coherent Gaussian beams that have the same beam waist diameter and beam waist position as the partially coherent beam but a smaller divergence; the divergence of the partially coherent beam is achieved by varying directions of the fully coherent beams that in combination result in a beam profile in the far field that has the defined divergence for the partially coherent beam. As representative beams, five  $M^2=1$  beams are shown.

The fully coherent beams are located “on top of each other” at the location of their beam waist, that is also the location of the beam waist of the GSB. In this way the superposition of the coherent beams produces the required beam waist diameter of the GSB. The divergence of the partially coherent beam (the GSB) is obtained by pointing the coherent beams (with the smaller divergence) in slightly different directions so that as a superposition, they make up a Gaussian irradiance profile for the freely propagating Gauss-Shell beam with the defined divergence. We see for this case that accommodation to the beam waist results in a retinal image profile which is also Gaussian and the image diameter and shape is not affected by the aperture. This can be understood because all of the fully coherent (lower divergence) beams are super-imposed at the location of the beam waist and all have the same irradiance profile there, which is at the same time the irradiance profile of the Gauss-Schell beam. Therefore “cutting” away some of the coherent beams (that at the aperture have a correspondingly smaller diameter as compared to the Gauss-Shell beam) for accommodation

to the beam waist, just results in a lower retinal irradiance overall, but the relative shape of the image is still a Gaussian profile as given by the beam waist as an optical “object” that is imaged onto the retina (the same way as for incoherent radiation a smaller pupil only reduces the irradiance level of the image but not the size and shape of the image). Accommodation to a position in front of or behind the beam waist presents a different situation, as indicated in Figure 3 above: for a large enough aperture where all the beams pass into the eye, the retinal image is equivalent to the irradiance present at the location of accommodation (for the example shown in the figure, five spots from the five beams shown). However, when some outer fully coherent beams are not passing through the aperture (for the example above, the smaller aperture only allows the central three beams to pass into the eye), they are also not available for the retinal image and the image is correspondingly smaller (for the example consisting of three spots). This example with five representative beams for the overall GSB is exactly the same situation as the example of the three collimated laser beams discussed in the ILSC 2015 paper [4].

Alternatively to the coherent beams with small divergence and varying directions, the Gauss-Schell beam can also be constructed by fully coherent beams that have the same divergence as the GSB but a correspondingly smaller beam waist (smaller by a factor  $M^2$ ). In this case, the large-divergence beams all point in the same direction (the direction of the axis of GSB, i.e. have parallel axis) and are “on top of each other” in the far field. At the location of the beam waist of the GSB, the beam waists of the fully coherent beams are laterally displaced (normal to the beam axis) so that overall, as superposition, the fully coherent beams make up the waist diameter and Gaussian irradiance profile defined for the GSB. In this case, for accommodation to the beam waist, each point of the beam waist can be considered to be imaged onto a corresponding (conjugate) point on the retina. Because the radiation fields (coherent beams) that are associated to the points of the beam waist make up the far-field irradiance profile that is incident on the eye (where they overlap), a smaller pupil means that less power per beam enters the eye but all beams do enter eye (i.e. part of the beams). As a consequence, for accommodation to the beam waist, the relative distribution of points in the image that is associated to the points in the beam waist of the GSB is not changed and again *for accommodation to the beam waist*, an aperture in the partially coherent beam does not change the shape and size of the image as compared to no aperture or a very large aperture. For the case of accommodation to a location in front of behind the beam waist, the rays that are imagined to make up the coherent beams (Figure 4) are, at the

position of the image, some distance apart and when the outer rays are blocked by the aperture, the image on the retina becomes smaller as compared to the case when all rays enter the eye.

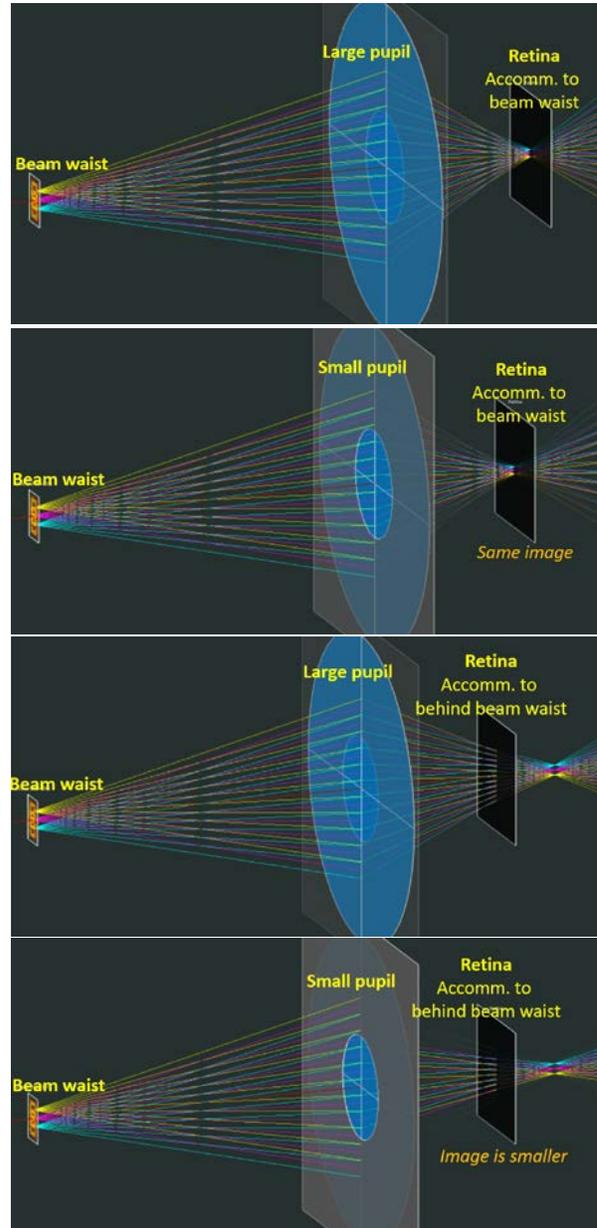


Figure 4. For the case of fully coherent beams with the same divergence as the GSB, the beam waists are small and the GSB at the beam waist location is constructed by positioning the fully coherent beams with lateral displacements. In this case three representative beams are shown being made up by rays that all passed into the eye for the large pupil, but the outer ones are blocked by the smaller pupil.

## Results for Two Exemplary Beams

The retinal image profile with and without an aperture on the imaging system was calculated for two exemplary beams. One has a relatively large  $M^2$  value of 300, the other of  $M^2 = 5$ . The  $M^2 = 300$  beam has a waist diameter of a little less than 1 mm (specified as the  $1/e^2$  diameter) and a relatively large divergence of 420 mrad. The two beams could for instance be the two axis of a line laser, but the two axis are here analyzed separately. The beam waist of the  $M^2 = 300$  beam is located at 90 mm from the eye and it is assumed that the eye can accommodate to that position. This is somewhat closer than is necessary to consider based on IEC 60825-1 where a minimum distance of accommodation of 100 mm is stated, but it helps as an example. As is consistent using the Gauss-Schell beam, the irradiance profile of the partially coherent beam is assumed to be Gaussian up to the aperture. For simplicity the distance between the imaging lens and the “retina” was set equal to 50 mm. This distance is not relevant as long as the characterisation of the retinal image is done in terms of angular subtense as seen from lens, i.e. dividing the image spatial distribution by 50 mm. The wavelength of the laser radiation was set to 905 nm.

Table 1. Input parameter of the sample beams.

		Collimated “high quality” beam	Diverging “low quality” beam
$M^2$		5	300
Waist diameter $1/e^2$	mm	2.2	0.81
Divergence, full angle, $1/e^2$	mrad	2.6	420
Raleigh length	mm	846	1.9
Distance from eye to beam waist	mm	10000	90
Beam diam. at 7 mm apert., $1/e^2$	mm	26	36

The location of the eye is in both beams well outside of the Raleigh length, so that the  $1/e^2$  diameter of the Gaussian beam profile that is incident on the aperture can simply be calculated by a multiplication of the divergence by the distance of the eye to the beam waist. It needs to be emphasized that a beam with the above listed properties most likely does not feature a Gaussian beam profile but could be “any” kind of profile when the beam diameter and divergence is determined according to ISO 11146. Both the beam profile that is incident on the “eye” can then be substantially different,

as well as the profile where the eye is accommodating to. However, the case of a Gaussian profile is still highly instructive to show the potential reduction of the retinal image as compared to the beam profile where the eye is accommodating to, when apertures are involved. This effect of the aperture was clearly not appreciated at the time when the beam propagation model was applied to laser safety in the late 1990s and is still sometimes overlooked or even disputed by some.

For both beams, the results of the calculations show that for accommodation to the position of the beam waist, there is no difference in the retinal image pattern when comparing the case of a 7 mm aperture with the case of an “infinite” aperture. The  $1/e$  diameter of the image (which is an option in IEC 60825-1 to be taken as criterion for  $\alpha$  for the case of Gaussian beam profiles; in this case the full power that passes through the 7 mm aperture is used as AE, not the partial power within the region defined by  $\alpha$  as border) in this case results in a value of  $\alpha$  that is equal to the angular subtense of the waist diameter (transformed into a  $1/e$  diameter value with the factor square-root of 2) as seen from the aperture of the eye.

The results of the modelling is shown in Table 2. For both beams, the angular subtense  $\alpha$  is shown for the case of “no aperture” and the 7 mm aperture. Profiles are shown in the figures further below. Accommodation to the beam waist was modelled, as well as accommodation to positions behind the beam waist specified as multiples of the Raleigh length  $z_R$ , as well as to infinity. The value of  $\alpha$  is given as  $1/e$  “diameter” as derived for the  $1/e^2$  which is the output of the model (i.e. reduction by  $\sqrt{2}$ ) with the exception of the values specified for the case of the 7 mm aperture and accommodation positions of 10 x the Raleigh or more behind the beam waist (the beam waist is at distance  $z_0$  seen from the eye, see Table 1), where the  $1/e^2$  value as given by the model is listed in the table. Since these profiles have relatively steep edges, the  $1/e$  and the  $1/e^2$  diameters are very close (for a top-hat profile, the  $1/e$  and the  $1/e^2$  diameter is the same).

Table 2. Angular subtense subtended by the retinal image ( $1/e$  diameter) calculated for no aperture (or an infinite aperture) and an aperture with 7 mm diameter on the imaging lens. The \* denotes that the image has relatively steep flanks and the original output of the model ( $1/e^2$  diameter) was not reduced.

	no aperture	7 mm aperture	
$M^2 = 300$	mrad	mrad	Ratio
beam waist	6.5	6.6	1.0
$z_0 + 0.25 \times z_R$	6.7	6.6	1.0
$z_0 + 0.5 \times z_R$	7.3	6.6	1.1
$z_0 + 1 \times z_R$	9.1	6.7	1.4
$z_0 + 2 \times z_R$	14.1	7.1	2.0
$z_0 + 5 \times z_R$	30.3	9.3	3.2
$z_0 + 10 \times z_R$	54.3	19.1*	2.8
$z_0 + 20 \times z_R$	91.8	28.8*	3.2
infinity	299.8	83.4*	3.6
	no aperture	7 mm aperture	
$M^2 = 5$	mrad	mrad	Ratio
beam waist	0.16	0.15	1.0
$z_0 + 0.25 \times z_R$	0.16	0.15	1.0
$z_0 + 0.5 \times z_R$	0.17	0.15	1.1
$z_0 + 1 \times z_R$	0.20	0.16	1.3
$z_0 + 2 \times z_R$	0.30	0.17	1.7
$z_0 + 5 \times z_R$	0.56	0.22	2.5
$z_0 + 10 \times z_R$	0.85	0.42*	2.0
infinity	1.82	0.77*	2.4

The data in Table 2 show that for accommodation to within a range of  $1/2$  of the Rayleigh length the retinal image for the case of an aperture is not more than 10% smaller as compared to the case without an aperture (where the retinal image is a direct image of the beam profile where the eye is accommodating to). For accommodation to the position in the beam equal to the Rayleigh length, the difference is about 1.4 (what could for a safety analysis be considered as significant when the AE is close to the AEL). As can be seen in Figures 5 and 6 below, the shape of the profile for the case of a 7 mm aperture remains Gaussian for up to about 5 times the Rayleigh length, and for 10 times the Rayleigh length deviates from a Gaussian and has a steeper edge (for  $M^2=300$  more significantly as for  $M^2=5$ ). For the case of accommodation to infinity, the retinal image with a 7 mm aperture has steep edges and the angular subtense of the image is, as expected (see also Figure 1) equal to

the angular subtense that the 7 mm aperture subtends as seen from the beam waist, i.e.  $7 \text{ mm} / z_0$ . For accommodation to infinity, the angular subtense of the image resulting with a 7 mm aperture is for the  $M^2 = 300$  beam a factor of 3.6 and for the  $M^2 = 5$  beam a factor of 2.4 smaller as compared to the  $1/e$  divergence for the two beams. Thus the general notion, that the value of  $\alpha$  is equal to the divergence is correct only for the case that the  $1/e$  beam diameter at the eye is not larger than 7 mm. When the  $1/e$  beam diameter is larger than the 7 mm aperture, the angular subtense of the retinal image will be limited to approximately  $7 \text{ mm} / z_0$ . However, accommodation to infinity is only relevant in the classification of a product for the case that the beam waist is relatively large and relatively close to the eye, so that accommodation to infinity results in the more restrictive retinal image as compared to accommodation to the beam waist.

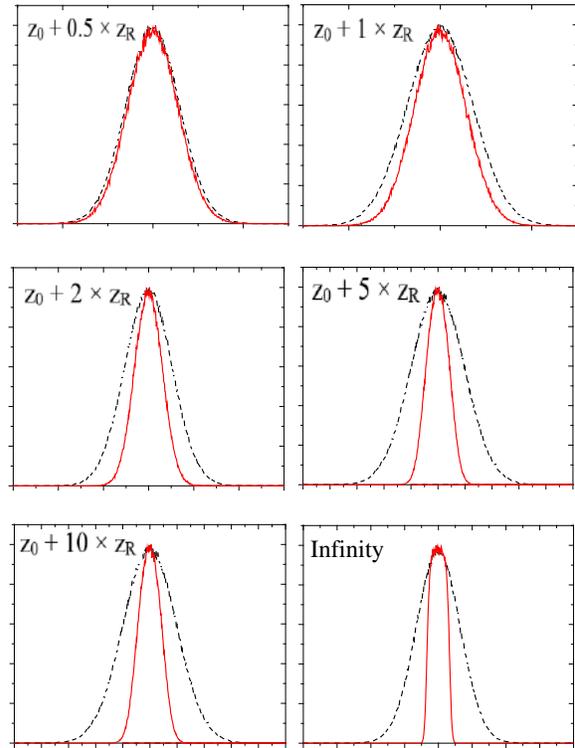


Figure 5. Retinal irradiance profiles (arbitrary scale for abscissa, irradiance ordinate scaled to 1) for the  $M^2=5$  beam. Plotted for a range of accommodation conditions, from top left: accommodation to half of the Rayleigh length behind the beam waist, to infinity. The full red line is the retinal profile when the beam passes through a 7 mm aperture at the imaging lens, the black dashed line is calculated for the case of an aperture that is larger than the beam.

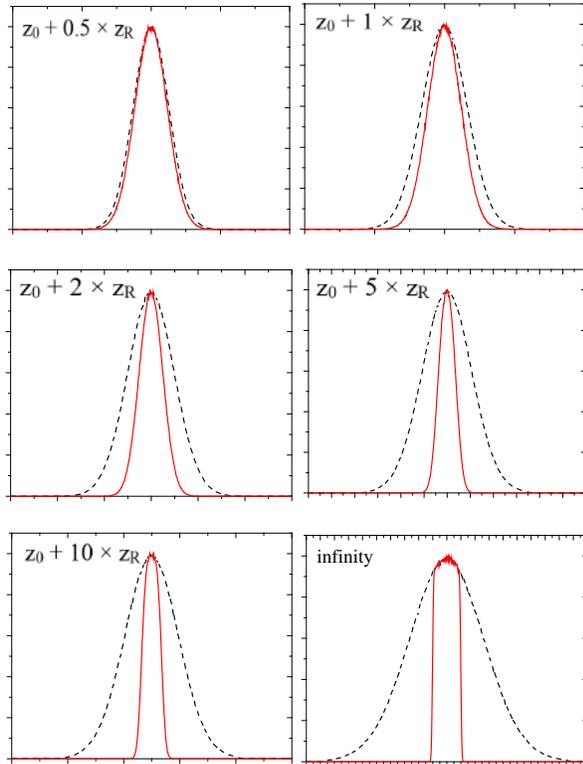


Figure 6. As in Figure 5 but for the  $M^2=300$  beam.

## Discussion

Based on the simulation results shown above, the impact of the 7 mm aperture to reduce the extent of the retinal image can be clearly seen for the case that the beam is larger than the 7 mm aperture. Only for accommodation to the beam waist and to within less than  $\frac{1}{2}$  of the Rayleigh length is the retinal image profile not affected by the 7 mm aperture. Only in this case can the profile where the eye is accommodating to be used directly as “apparent source” to determine the image of the apparent source. As is shown by Kotzur et al. in these proceedings [9], the truncation of the image by the 7 mm aperture also applies to fully coherent beams with  $M^2 = 1$ .

For the two beams modelled, if they were a stigmatic beam, accommodation to the beam waist produces the smallest value of  $\alpha$  (however, for the well collimated beam of  $M^2 = 5$  the angular subtense is in all cases, for all accommodation conditions to between the beam waist and infinity, smaller than  $\alpha_{\min}$ ). For the example of the  $M^2 = 300$  beam, if it were a stigmatic beam (circularly symmetric), accommodation to the beam waist is associated to the smallest  $\alpha$  and for accommodation to the beam waist, the 7 mm aperture

does not change the relative image profile; the value of  $\alpha$  with the 1/e diameter criterion equals 6.6 mrad.

As commented in the first part of the paper, even a low value of  $M^2 = 5$  or a fully coherent beam with  $M^2 = 1$  can be associated to an extended source, when it is one axis of an astigmatic beam where the beam waists are located sufficiently apart (at different distances from the eye  $z_0$ ). For instance, when the two modelled beams were characterizing the two axis of an astigmatic beam, accommodation to the beam waist of the  $M^2=300$  beam would be the most restrictive accommodation overall and at that position, the collimated beam has a 1/e<sup>2</sup> diameter of 26 mm and about 18 mm when stated as 1/e (with  $\alpha$  larger than 200 mrad for the case of an infinite aperture on the imaging optics as seen from 90 mm). However, the collimated axis is heavily affected by the truncation of the beam with the 7 mm aperture and the resulting image width in the collimated direction is defined by the 7 mm aperture, i.e. as if the 7 mm aperture cuts out the central part of the collimated beam (resulting in sharp edges of the retinal image, similar to accommodation to infinity shown in Figure 5) that is then also relevant as “apparent source” for accommodation to 90 mm in front of the eye, resulting in an angular subtense of a little more than 70 mrad (7 mm divided by distance of accommodation position to the eye) instead of more than 200 mrad for the case of a very larger aperture.

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# Notes on the determination of the angular subtense of the apparent source in laser safety

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