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Beam Propagation Hazard Calculations for Telescopic Viewing of Laser Beams

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Reference information for this proceedings paper

Title: *Beam Propagation Hazard Calculations for Telescopic Viewing of Laser Beams*

Authors: *Grabner U, Vees G, Schulmeister K*

Proceeding of the International Laser Safety Conference, March 10-13th 2003
Jacksonville, Florida, USA
Page 116-125

Published by the Laser Institute of America, 2003
Orlando, Florida, USA www.lia.org

Beam Propagation Hazard Calculations for Telescopic Viewing of Laser Beams

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ABSTRACT

We have performed beam propagation calculations which can be used to characterize the retinal spot size of laser beams for telescopic viewing. The results show that the minimal retinal spot size produced on the retina by using a telescope compared to that of the naked eye is increased at least by a factor that is equal to the magnifying power of the telescope and an increase of C_6 (or C_E) is applicable.

The hazard evaluation method used is based on the calculation of the maximum level of thermal hazard that is defined as the ratio of the power that enters the eye and the retinal spot diameter. Beam propagation principles were used to calculate the distance between the beam waist and the eye lens that provides the maximum hazard. The only two input parameters necessary for these calculations are the waist diameter and the far-field divergence of the beam. The beam width on the retina at the most hazardous viewing distance is consequently used to determine the angular subtense of the apparent source that is needed to calculate the MPEs and AEL values. Both the angular subtense of the apparent source and the most hazardous viewing distance have been calculated for a wide range of beam waist widths and far-field divergences.

1. INTRODUCTION

The correction factor C_6 that is contained in the MPEs for retinal thermal injury characterises that the hazard from extended sources is lower than from collimated laser beams or other point sources.[1,2] The factor C_6 depends on the plane angle α subtended by the diameter of the retinal image at the lens of the eye and measured in mrad. This angle is equivalent to the size of the image on the retina and this in turn is directly related to the source size as shown in figure 1. In the safety standards, a minimal angular subtense, α_{min}, is defined as 1.5 mrad, an angle that corresponds to a retinal diameter of 25 µm for a focal length of the eye of 17 mm.

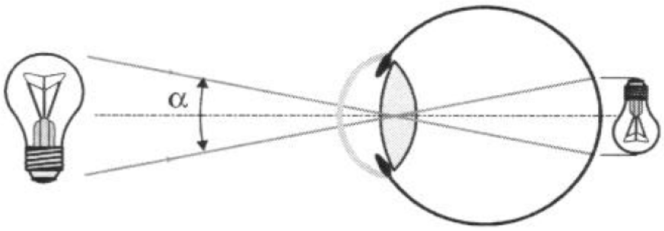

Figure1: The angular subtense α of an object

For laser radiation the angular subtense α of a source is often referred to as the apparent source size, to denote that it is not related to the physical dimension of the emitter such as the beam diameter or the exit mirror. The size and location of the apparent source, which is defined as the *real or virtual object that forms the smallest possible retinal image by eye focusing* [2], can be thought of as the size and the location of a conventional source, which leads to the same image size on the retina. As the angular subtense of the apparent source is directly related to the diameter of the irradiated area on the retina, together with the energy or power which is passing through the pupil and is incident on the retina these two parameters determine the exposure or irradiance at the retina. Generally, for a given energy or power, an increase in image size reduces the retinal exposure (J/m^2) and consequently decreases the injury potential.

The correction factor C_6 is defined in the IEC 60825-1 and ANSI Z136.1 as being the ratio of the angular extent α of the apparent source to α_{min}:

$$C_6 = \frac{\alpha}{\alpha_{min}} \qquad (1)$$

C_6 is limited to values between 1 and 66.67, as for α greater 100 mrad the dependence of the heat flow on the image size does no longer apply and the limit could be expressed as constant radiance limit, which is reflected by setting the image size constant as well as by specifying a field of view of 100 mrad for the irradiance measurement, which limits the measured part of the source to 100 mrad.

Based on the dependence of the thermal retinal MPE on α, we have developed a beam propagation model to characterize the hazard from exposure to laser beams while using 7 x 50 binoculars.

2. THERMAL HAZARD

Following the linear dependence of thermal injury on the angular subtense of the source, which is again a linear function of the image diameter on the retina, the quantity that describes the level for thermal hazard, is the beam power or energy that falls on the retina divided by its width at the retina. Since, for a given wavelength, the beam power that is incident on the retina is proportional to the power that enters the eye P_r (in the sense of the part of the power in beam that is incident on the eye and that passes through the pupil, usually referred to as intraocular power), the calculations are based on this value. The thermal hazard H can then be defined as:

$$H = \frac{P_r}{d_r} \tag{2}$$

where d_r is the width or the diameter of the beam at the retina. Following the concept developed by Brooke Ward, the maximum hazard for retinal damage is found by maximizing the parameter H. [3]

3. HUMAN EYE MODEL

Refraction in the eye takes place mostly at the cornea whereas the eye lens serves the purpose of focusing (accommodation). The range of accommodation of the eye varies greatly from one person to another and in each person, with age. It is maximum for young people and reduces by age. [4]

The eye model used in this analysis consists of a single thin lens with a variable focal length, combining the refraction on the cornea and the eye lens, and a fixed lens to retina spacing of 17 mm. Although this is a strong simplification, it describes the complexity of the eye in a sufficient manner. To consider the accommodation range of most people, a standard eye is used having a focal length in a range of $f_{e,min}$ = 14.53 mm to $f_{e,max}$ = 17 mm, giving a minimum and maximum focusable distance of 100 mm and infinity, respectively. The pupil diameter in this eye model is fixed to 7 mm.

4. BEAM PROPAGATION

The propagation of an ideal TEM_{00} beam in free space is described by the following equations:

$$w_0(z) = w_w \cdot \left(1 + \frac{z^2}{z_R^2}\right)^{\frac{1}{2}} \tag{3}$$

$$R(z) = z + \frac{z_R^2}{z} \tag{4}$$

$$z_R = \frac{\pi \cdot w_w^2}{\lambda} \tag{5}$$

$$\theta = \frac{w_w}{z_R} \tag{6}$$

$$I(r,z) = I_0(z) \cdot e^{-\frac{2 \cdot r^2}{w_0^2(z)}} \tag{7}$$

where $w_0(z)$ is the beam radius within which 86.46% of the total energy/power is enclosed and the intensity has reduced to $1/e^2$ of the peak intensity, w_w is the beam waist radius, $R(z)$ is the curvature of the wave front at distance z, z_R is the Rayleigh length, λ is the wavelength of the radiation and θ the far-field divergence.

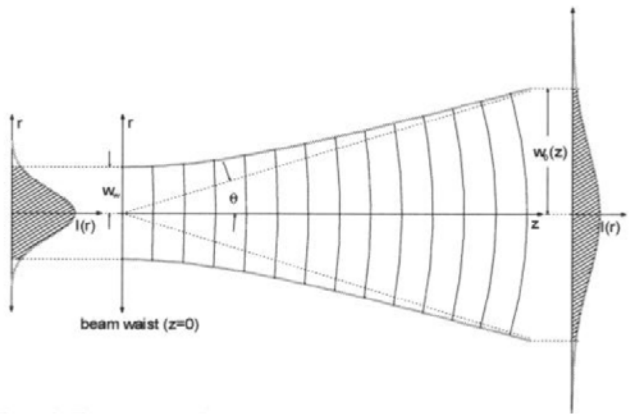

Figure 2. Characteristics of a gaussian beam

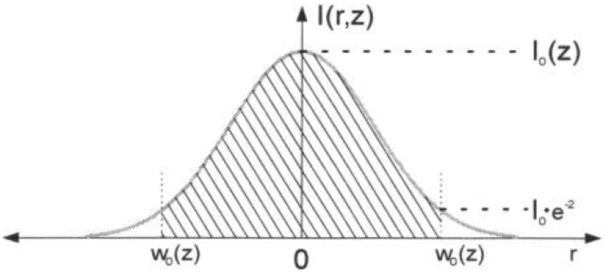

Figure 3: Radial intensity distribution of gaussian beam

The TEM_{00} beam is only the lowest-order solution in free space. In general, a laser beam consists of further higher modes. In this case the equations 3, 4, 6 and 7 are still applicable, but the formula for the Rayleigh length has to be modified to

$$z_R = \frac{1}{M^2} \cdot \frac{\pi \cdot w_w^2}{\lambda} \tag{8}$$

M^2 is the beam propagation ratio, which is defined using an invariant of propagation that is not changed by any linear, diffraction limited optical system: the product of the beam waist radius w_w and the far-field divergence. M^2 is defined as the ratio of the value of this invariant for the real beam to its value for a TEM_{00} beam of the same wavelength. M^2 is equal to 1 for TEM_{00} beams and greater than 1 for higher order modes.[5]

$$M^2 = \cdot \frac{w_w \ \cdot \ \theta}{w_{w,TEM_{00}} \cdot \theta_{TEM_{00}}} \tag{9}$$

4.1 Transformation of a gaussian beam by any diffraction limited optical system

To describe the propagation of a laser beam through any optical system the complex beam parameter $q(z)$ was used, which for a TEM_{00} beam is defined as. [6]

$$\frac{1}{q(z)} = \frac{1}{R(z)} - i \cdot \frac{\lambda}{\pi \cdot w_0^2(z)} \tag{10}$$

$q(z)$ is a complex function and part of the lowest-order solution of the paraxial Helmholtz equation

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} - 2ik \cdot \frac{\partial U}{\partial z} = 0 \tag{11}$$

for the complex field amplitude

$$\underline{U}(r,z) = \hat{E} \cdot e^{-i\left\{p(z) + k \cdot r^2 / 2 \cdot q(z)\right\}} \tag{12}$$

of a rotationally symetric electrical field

$$\underline{E}\left(\vec{r}\right) = U\left(r,z\right) \cdot e^{-ikz} \tag{13}$$

The relation between the complex functions $q(z)$ and $p(z)$ can be derived by inserting Equ. 12 into Equ. 11

$$\frac{\partial p(z)}{\partial z} = -\frac{i}{q(z)} \qquad \frac{\partial q(z)}{\partial z} = 1 \tag{14}$$

Inserting Equ. 10 into Equ. 12 gives the solution for the electrical field of a TEM_{00} beam

$$\underline{E}(x,y,z) = \hat{E} \cdot e^{-r^2 / w_0^2(z)} \cdot e^{-ik \cdot r^2 / 2 \cdot R(z)} \cdot e^{-i[kz + p(z)]} \tag{15}$$

where

$k \cdot r^2 / 2 \cdot R(z)$ is the transversal phase factor

and

$p(z) = \arctan\left(\frac{z}{z_R}\right)$ is the longitudinal phase factor of the gaussian wave.

As the wave front at the beam waist is planar ($R(z = 0) \rightarrow \infty$), a beam waist is assumed to be located at $z = 0$. Considering

$$q(z=0) = q_w \quad \text{and} \quad \frac{1}{q(z=0)} = -i \cdot \frac{\lambda}{\pi \cdot w_0^2 (z=0)} = -i \cdot \frac{\lambda}{\pi \cdot w_w^2} \tag{16}$$

the complex beam parameter of the beam waist can be evaluated:

$$q_w = i \cdot \frac{\pi \cdot w_w^2}{\lambda} = i \cdot z_R \tag{17}$$

Integrating Equ. 14 gives the complex beam parameter

$$q(z) = z + q_w = z + i \cdot z_R \tag{18}$$

$q(z)$ is a linear function of z and transforms in the same way as the radius of curvature of spherical waves coming from a point source

$$q_2 = \frac{A \cdot q_1 + B}{C \cdot q_1 + D} \qquad \text{ABCD-law for gaussian beams} \tag{19}$$

where q_1 is the complex beam parameter at $z = z_1$, q_2 is the complex beam parameter at $z = z_2$ and

$$\mathbf{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{20}$$

is the matrix of the optical system.

4.1.1 Transformation of a gaussian beam by a thin lens

Assuming a gaussian beam with a waist radius w_{w1} at $z = 0$ in front of a thin lens, that has a focal length f and is located at $z = d_1$, leads to figure 4:

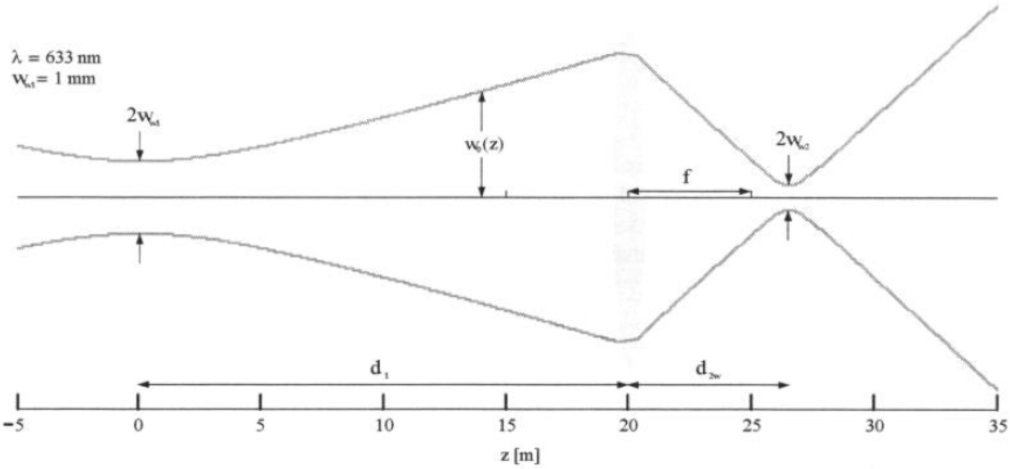

Figure 4: Transformation of a gaussian beam by a thin lens

To obtain the complex beam parameter $q_2 = q\,(z = d_1 + d_2)$ at the distance d_2 behind the lens the matrix **M** has to be evaluated

$$\mathbf{M} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & -d_2 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \otimes \begin{pmatrix} 1 & -d_1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{d_2}{f} & -d_1 - d_2 \cdot \left(1 - \frac{d_1}{f}\right) \\ \frac{1}{f} & 1 - \frac{d_1}{f} \end{pmatrix} \tag{21}$$

Inserting

$$q_1(z=0) = q_{w1} = i \cdot z_{R1} = i \cdot \frac{\pi \cdot w_{w1}^2}{\lambda} \tag{22}$$

into Equ. 19 the following equation for q_2 can be found:

$$q_2\left(d_1+d_2\right)=\frac{i\cdot Az_{R1}+B}{i\cdot Cz_{R1}+D}=\frac{ACz_{R1}^2+BD+i\cdot z_{R1}\left(AD-BC\right)}{C^2z_{R1}^2+D^2}=\frac{BD+i\cdot z_{R1}}{D^2}=$$

$$=\frac{\left(1-\frac{d_2}{f}\right)\cdot\frac{1}{f}\cdot z_{R1}^2+\left(-d_1-d_2\cdot\left(1-\frac{d_1}{f}\right)\right)\cdot\left(1-\frac{d_1}{f}\right)+i\cdot\left(\left(1-\frac{d_2}{f}\right)\cdot\left(1-\frac{d_1}{f}\right)-\left(-d_1-d_2\cdot\left(1-\frac{d_1}{f}\right)\right)\frac{1}{f}\right)\cdot z_{R1}}{\left(\frac{1}{f}\right)^2\cdot z_{R1}^2+\left(1-\frac{d_1}{f}\right)^2} \tag{23}$$

Assuming that a new beam waste with radius w_{w2} is formed at $z = d_1 + d_{2w}$ results in

$$q_2\left(d_1+d_{2w}\right)=q_{w2} \tag{24}$$

with

$$\mathrm{Re}\left[q_{w2}\right]=0=\left(1-\frac{d_{2w}}{f}\right)\cdot\frac{1}{f}\cdot z_{R1}^2+\left(-d_1-d_{2w}\cdot\left(1-\frac{d_1}{f}\right)\right)\cdot\left(1-\frac{d_1}{f}\right) \tag{25}$$

Solving Equ. 25 leads to d_{2w} as a function of d_1, z_{R1} and f:

$$d_{2w}=f+\left(V\left(f,d_1\right)\right)^2\cdot\left(d_1-f\right) \tag{26}$$

where

$$V\left(f,d_1\right)=\frac{f}{\sqrt{\left(f-d_1\right)^2+z_{R1}^2}} \tag{27}$$

Inserting d_{2w} into Equ. 23 gives z_{R2} and w_{w2}:

$$\mathrm{Im}\left[q_{w2}\right]=z_{R2}=\frac{\pi\cdot w_{w2}^2}{\lambda} \tag{28}$$

$$w_{w2}=\frac{f}{\sqrt{\left(f-d_1\right)^2+z_{R1}^2}}\cdot w_{w1}=V\left(f,d_1\right)\cdot w_{w1} \tag{29}$$

4.1.2 Transformation of a gaussian beam by a telescope

The beam propagation model was developed for an astronomical refracting telescope that consists of two convergent lenses. The results of the simulation are however also applicable to a binocular with corresponding focal lengths. In the normal state of adjustment the second focal plane of the first lens (the objective), coincides with the first focal plane of second lens (the eyepiece), so that an incident pencil of parallel rays emerges as a parallel pencil. [4]

For the calculations, the waist of a gaussian beam with a waist radius w_{w1} is assumed to be at $z = 0$. The objective of the telescope with a focal length f_1 is located at $z = d_1$ at a distance $f_1 + f_2$ in front of the eyepiece with a focal length f_2. To receive the complex beam parameter $q_2 = q\,(z = d_1 + f_1 + f_2 + d_2)$ at distance d_2 from the lens the matrix **M** has to be calculated

$$\mathbf{M}=\begin{pmatrix}-\frac{f_2}{f_1} & d_1\cdot\frac{f_2}{f_1}+d_2\cdot\frac{f_1}{f_2}-\left(f_1+f_2\right)\\ 0 & -\frac{f_1}{f_2}\end{pmatrix} \tag{30}$$

Doing calculations similar to that done in the case of the thin lens d_{2w} can be evaluated as a function of d_1, f_1 and f_2:

$$d_{2w}=-d_1\cdot\left(\frac{f_2}{f_1}\right)^2+\left(f_1+f_2\right)\cdot\left(\frac{f_2}{f_1}\right) \tag{31}$$

It should be noted that d_{2w} does not depend on z_{R1}, and has a maximum for $d_1 = 0$ which is equal to the distance between the exit pupil and the eyepiece. d_{2w} can also be negative, creating a virtual beam waist inside the telescope:

$$d_{2w\ \max} = (f_1 + f_2) \cdot \left(\frac{f_2}{f_1}\right) \qquad \text{for} \quad d_1 = 0$$

$$d_{2w} \leq 0 \qquad \text{for} \quad d_1 \geq (f_1 + f_2) \cdot \frac{f_1}{f_2} \tag{32}$$

z_{R2} and w_{w2} can be derived to:

$$z_{R2} = \frac{\pi \cdot w_{w2}^2}{\lambda} = \left(\frac{f_2}{f_1}\right)^2 \cdot z_{R1} \tag{33}$$

$$w_{w2} = \left(\frac{f_2}{f_1}\right) \cdot w_{w1} \tag{34}$$

Note that the radius of the beam waist produced by the telescope is not a function of d_1 and depends only on the ratio of f_1 and f_2, which is defined as the magnifying power of the telescope.

Equ. 26, 27, 28, 29 and Equ. 31, 33, 34 have been derived considering a TEM_{00} beam. Again in case of a real laser beam these equations will still be applicable, if the formula for the Rayleigh length is modified to

$$z_R = \frac{1}{M^2} \cdot \frac{\pi \cdot w_w^2}{\lambda}$$

5. DETERMINATION OF THE MINIMAL RETINAL SPOT SIZE

The eye model used in the following analysis consists of a single thin lens with a variable focal length and an image plane (retina) fixed behind the lens at a constant distance of d_e = 17mm from the lens. This case seems quite similar to that of a single thin lens with a fixed focal length producing a beam waist in a distance that can be determined by Equ. 26. Hence one could presume that the minimal spot size is a beam waist that is located at the retina, but this is not the case. In general, to produce a minimal spot size on the retina, a beam waist is formed in a very short distance in front of the retina, resulting in a lens focal length that is different to that forming a beam waist on the retina.

To minimize the spots size on the retina an equation for the beam propagation behind the eye lens has to be derived. Taking into account that the new beam waist is located at the position $z = d_1 + d_{2w}$, Equ. 26 and Equ. 29 can be inserted into

$$w_{02}(z) = w_{02}(w_{w1}, d_1, f, z) = w_{w2}\sqrt{1+\left(\frac{z-(d_1+d_{2w})}{z_{R2}}\right)^2} \tag{35}$$

By setting $z' = z - d_1$, the beam radius behind the eye lens can be evaluated as a function of the distance from the eye lens.

$$w_{02}(w_{w1}, d_1, f_e, z') = \frac{w_{w1} \cdot z'}{z_{R1} \cdot \left(1+\frac{d_1^2}{z_{R1}^2}\right)^{\frac{1}{2}}} \cdot \sqrt{1 + z_{R1}^2 \cdot \left(1+\frac{d_1^2}{z_{R1}^2}\right)^2 \cdot \left(\frac{1}{z'} - \frac{1}{f_e} + \frac{1}{d_1 + \frac{z_R^2}{d_1}}\right)^2} = \tag{36}$$

Minimizing Equ. 36 for given values of w_{w1}, d_1 and $z' = d_e$ = 17 mm gives us an equation for the eye focal length f_{me}, that produces the minimal beam diameter on the retina

$$\frac{1}{d_e} - \frac{1}{f_{me}} + \frac{1}{d_1 + \frac{z_{R1}^2}{d_1}} = 0 \qquad \Rightarrow \qquad f_{me} = \frac{1}{\frac{1}{d_e} + \frac{1}{R_1(d_1)}} = \frac{d_e(d_1^2 + z_{R1}^2)}{d_1^2 + z_{R1}^2 + d_1 \cdot d_e} \tag{37}$$

where $R_1(d_1)$ is the radius of curvature of the input beam as a function of the distance d_1 from its beam waist.

If f_{me} is within the eye accommodation range the minimal spot size obtainable by eye focusing is

$$w_{me}(w_{w1},d_1) = w_e(w_{w1},d_1,\, f_e = f_{me}) = \frac{w_{w1}\cdot d_e}{\left(d_1^2 + z_{R1}^2\right)^{1/2}} = \frac{\lambda\cdot d_e}{\pi\cdot w_{01}(d_1)} \tag{38}$$

where $w_{01}(\, d_1\,)$ is the radius of the input beam at the lens:

$$w_{01}(d_1) = w_{w1}\cdot\left(1+\frac{d_1^2}{z_{R1}^2}\right)^{1/2} \tag{39}$$

Hence the beam radius behind the lens as a function of the distance z' from the lens can be calculated with

$$w_e(z') = \frac{\lambda\cdot z'}{\pi\cdot w_{01}(d_1)}\cdot\sqrt{1+\frac{w_{01}^4(d_1)\cdot\pi^2}{\lambda^2}\cdot\left(\frac{1}{z'}-\frac{1}{d_e}\right)^2} \tag{40}$$

It should be noted that as long as the eye is able to accommodate, two different gaussian beams with equal wavelengths and the same beam diameters on the lens are transformed to identically propagating beams behind the eye lens, however, the eye focal length is different in each case.

So far, the calculations did not take into account that the focal length f_{me} has to be within the accommodation range of the eye. As it can be seen in Equ. 36 the spot radius on the retina $w_e\,(\,f_e\,)$ as function of the eye focal length has only one minimum at $f_e = f_{me}$. Consequently it is an increasing function for $f_e > f_{me}$ and a decreasing function for $f_e < f_{me}$. Therefore in the case of $f_{me} < f_{e,min}$ = 14.53 mm the minimum spot radius achievable by eye focusing is obtained with $f_e = f_{e,min}$ whereas in the case of $f_{me} > f_{e,max}$ = 17 mm the minimum spot radius is obtained with $f_e = f_{e,max}$.

$$w_{me} = \frac{w_{w1}\cdot d_e}{\left(d_1^2 + z_{R1}^2\right)^{1/2}}\cdot\sqrt{1+\frac{d_1^4}{z_{R1}^2}\cdot\left(1+\frac{z_{R1}^2}{d_1^2}\right)^2\cdot\left(\frac{1}{d_e}-\frac{1000}{14.53}+\frac{1}{d_1+\frac{z_{R1}^2}{d_1}}\right)^2} \qquad \text{if } f_{me} < f_{e,min} = 14.53\text{ mm}$$

$$w_{me} = \frac{w_{w1}\cdot d_e}{\left(d_1^2 + z_{R1}^2\right)^{1/2}} \qquad \text{if } 14.53\text{ mm} \le f_{me} \le 17\text{ mm} \tag{41}$$

$$w_{me} = \frac{w_{w1}\cdot d_e}{\left(d_1^2 + z_{R1}^2\right)^{1/2}}\cdot\sqrt{1+\frac{d_1^4}{z_{R1}^2}\cdot\left(1+\frac{z_{R1}^2}{d_1^2}\right)^2\cdot\left(\frac{1}{d_e}-\frac{1000}{17}+\frac{1}{d_1+\frac{z_{R1}^2}{d_1}}\right)^2} \qquad \text{if } f_{me} > f_{e,max} = 17\text{ mm}$$

5.1. Minimal retinal spot size in case of telescopic viewing

The telescope used in the following calculations is an astronomical refracting telescope with a focal length of the objective of f_1 = 14 cm, a focal length of the eyepiece of f_2 = 2 cm and a distance between the two lenses of $d_T = f_1 + f_2$ = 16 cm, providing a magnifying power of 7. For these calculations the diameter of the lenses are not relevant. To obtain the minimal spot size on the retina in case of using a telescope, Equ. 31, 33 and 34 have to be combined with Equ. 41. In addition it has to be taking into account that the eye lens is located at the exit pupil of the telescope at a distance $d_{EP} = f_2\cdot(\,f_1 + f_2\,)\,/\,f_1$ behind the eyepiece, to ensure that all the light entering the objective at different off-axis angles will reach the eye.

Figure 5 shows the minimal spot size of the naked eye w_{me} for a number of beams with a common waist radius of 1 mm in comparison to that produced on the retina by using the telescope w^T_{me} for distances d between the input beam waist and the position of the objective up to 30 m. At large distances the ratio $w^T_{me}\,/\,w_{me}$ converges to 7, the value of the magnifying power of the telescope, whereas if d becomes smaller, the ratio at first reaches a maximum to decrease again for even closer distances to a value of almost 7 for d = 0. It has to be reemphasised, that the ratio is always bigger than 7.

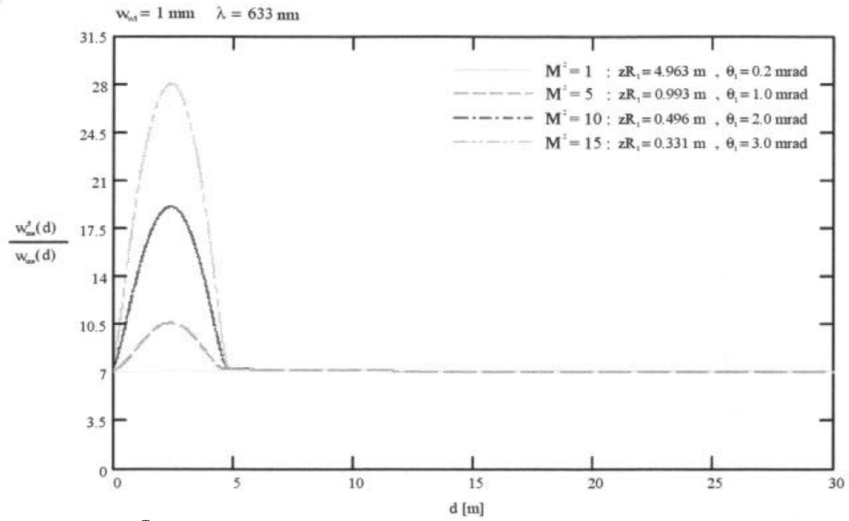

Figure 5: Ratio w^T_{me} / w_{me} of the minimal retinal beam radii with and without telescopic viewing

6. MAXIMUM THERMAL HAZARD, MOST HAZARDOUS DISTANCE AND ANGULAR SUBTENSE

Assuming a gaussian power distribution, we obtain the following expression for the power $P(u)$ transmitted through a pupil with diameter u

$$P(u) = P_{tot} \cdot \left(1 - e^{-\frac{2\cdot u^2}{w_0^2}}\right) \qquad (42)$$

where P_{tot} is the total beam power and w_0 is the $1/e^2$ beam radius at the pupil.
It should be mentioned that the use of Equ. 42 in the following calculations will overestimate the level of hazard as the gaussian distribution provides the greatest threat compared to other distributions and as effects of diffraction on the beam diameter at the pupil are neglected.[3]

6.1 Naked eye

In the case of the human eye the pupil diameter is 7 mm. If a laser with a certain beam diameter and far-field divergence moves away from the eye, the diameter on the lens increases reducing the transmitted power incident on the retina. At the same time the minimal spot size on the retina decreases and there is a certain distance where the thermal hazard H becomes a maximum. This distance can be referred to as the most hazardous viewing distance. To obtain the corresponding angular subtense, the 1/e spot diameter at the retina has to be divided by the lens-to-retina spacing. For the naked eye, the most hazardous distance and the corresponding angular subtense have been calculated by Brooke Ward for a large range of beam characteristics. Beams with a far-field divergence up to 400 mrad and waist widths up to 6 mm have been analysed to reveal the locations of the peak hazard as well as the value of the beam width at the retina at that viewing condition. One of the results was that there exists a range of low divergence beams where the most hazardous viewing distance is significantly greater than the standard of 100 mm. In contrast to this there are also some high divergence source conditions that represent the highest hazard when placed closer than 100 mm from the eye, despite the eye is outside its accommodation range.[3]

6.2 Telescope

In the case of using a telescope it has to be taken into account that there is another aperture at the objective and at the eyepiece, which additionally truncates the input beam. Considering this, the power entering the eye can be evaluated with

$$P_r = P_{tot} \cdot \left(1 - e^{-\frac{2\cdot D_{OB}^2}{w_{OB,0}^2}}\right) \cdot \left(1 - e^{-\frac{2\cdot D_{EP}^2}{w_{EP,0}^2}}\right) \cdot \left(1 - e^{-\frac{2\cdot {D_{EL}^2}^2}{w_{EL,0}^2}}\right) \qquad (43)$$

where D_{OB} , $w_{OB,0}$, D_{EP} , $w_{EP,0}$, D_{EL} and $w_{EL,0}$ are the radii of and the $1/e^2$ beam radii at the objective, the eyepiece and the eye pupil, respectively.

The telescope used in the following analysis has an objective focal length of f_1 = 14 cm, a focal length of the eyepiece of f_2 = 2 cm and a distance between the two lenses of $d_T = f_1 + f_2$ = 16 cm, as in the calculations for figure 5. While the objective diameter has to be 50 mm to ensure an exit pupil of the same size as the eye pupil, the diameter of the eyepiece was chosen to be 25 mm.
The most hazardous viewing distance and the corresponding angular subtense were calculated for telescopic viewing of a laser beam. To account for a minimal 1/e beam diameter of 25 µm on the retina, the beam diameter was restricted to that value during the entire evaluation. The results are shown in the figures 6, 7, 8 and 9.

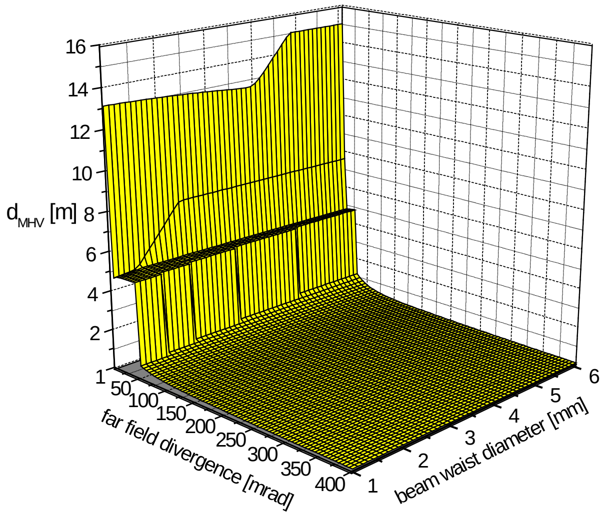

Figure 6: Most hazardous viewing distance d_{MHV} as a function of the waist diameter and divergence of the beam

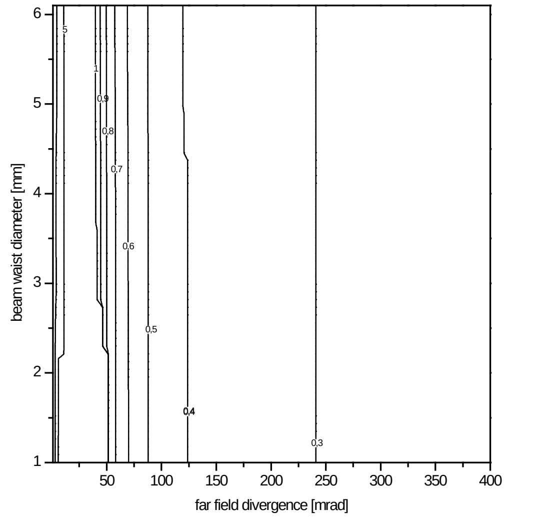

Figure 7: Contours of the most hazardous viewing distance d_{MHV} [m] as a function of the waist diameter and divergence of the beam

As it can be seen in figures 6 and 7, the most hazardous viewing distance as well as the relaxation factor C_6 mainly depend on the far-field divergence. For beams with a far-field divergence of more than 50 mrad the most hazardous viewing distance lies between 22 cm and 1 m. That means that the beam waist is located close to the objective of the telescope. If the far-field divergence becomes smaller than 50 mrad the most hazardous viewing distance rapidly increases up to a value of about 15 m for a beam with a very low divergence and a small waist diameter. This is because the power transmitted through the telescope and the eye pupil decreases much less than the spot size on the retina.

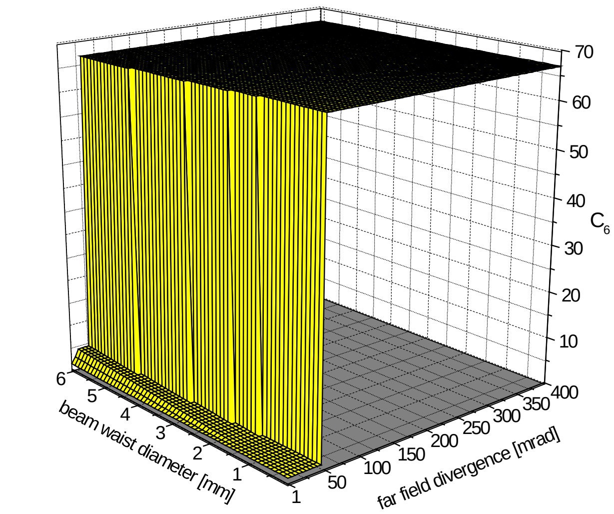

Figure 8: Thermal Relaxation factor C_6 as a function of the waist diameter and divergence of a beam

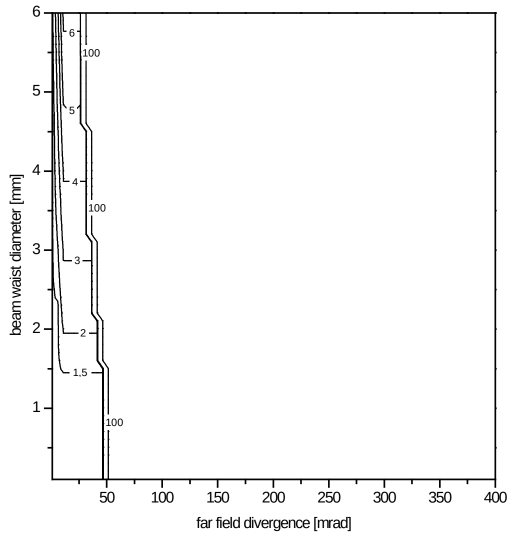

Figure 9: Contours of the angular subtense of the apparent source as a function of the waist diameter and divergence of a beam

50 mrad also marks a significant border line in figures 8 and 9. All beams with a far-field divergence of more than 50 mrad produce beam diameters at the retina that correspond to an angular subtense equal or even bigger

than the maximum α_{max} = 100 mrad whereas the angular subtense for beams with a far-field divergence between 1 mrad and 50 mrad does not exceed 7 mrad.

7. CONCLUSIONS

The calculations presented above show that the minimal retinal spot size produced on the retina by using a telescope compared to that of the naked eye is increased by a factor that is at least equal to the magnifying power of the telescope.

In addition it could be demonstrated that even in the case of telescopic viewing, the angular subtense of the apparent source can be obtained by the knowledge of the waist diameter and the far-field divergence of the beam. The analysis of the most hazardous viewing distance and the corresponding angular subtense included a wide range of beam waist diameters (1 mm to 6 mm) and far-field divergences (1 mrad to 400 mrad) covering beam propagation ratios M^2 up to 3000. It was shown, that both the most hazardous viewing distance and the angular subtense are primarily functions of the far-field divergence. The computations reveal that all beams with a field divergence of more than 50 mrad have an angular subtense of α_{max} = 100 mrad whereas the angular subtense for beams with far-field divergences between 1 mrad and 50 mrad does not exceed 7 mrad. The analysis also indicates that the most hazardous viewing distance for beams with a field divergence above 50 mrad is less than 1 m.

8. REFERENCES

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